Diagnostic Checking for Weibull Autoregressive Conditional Duration Models

Yao Zheng, Yang Li, Wai Keung Li and Guodong Li

Abstract We derive the asymptotic distribution of residual autocorrelations for the Weibull autoregressive conditional duration (ACD) model, and this leads to a portmanteau test for the adequacy of the fitted Weibull ACD model. The finite-sample performance of this test is evaluated by simulation experiments and a real data example is also reported.

Keywords Autoregressive conditional duration model · Weibull distribution · Model diagnostic checking · Residual autocorrelation

Mathematics Subject Classification (2010) Primary 62M10 · 91B84; Secondary 37M10

1 Introduction

First proposed by Engle and Russell [3], the autoregressive conditional duration (ACD) model has become very popular in the modeling of high-frequency financial data. ACD models are applied to describe the duration between trades for a frequently traded stock such as IBM and it provides useful information on the intraday market activity. Note that the ACD model for durations is analogous to the commonly used generalized autoregressive conditional heteroscedastic (GARCH) model [1, 2] for stock returns. Driven by the strong similarity between the ACD and GARCH models,

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428552_1_En_4_Chapter ☑ TYPESET ☐ DISK ☐ LE ☑ CP Disp.:29/8/2016 Pages: 9 Layout: T1-Standard

various extensions to the original ACD model of Engle and Russell [3] have been suggested. However, despite the great variety of ACD specifications, the question of model diagnostic checking has received less attention.

The approach used by Engle and Russell [3] and widely adopted by subsequent authors to assess the adequacy of the estimated ACD model consists of applying the Ljung–Box Q-statistic [7] to the residuals from the fitted time series model and to its squared sequence. The latter case is commonly known as the McLeod–Li test [8]. As pointed out by Li and Mak [5] in the context of GARCH models, this approach is questionable, because this test statistic does not have the usual asymptotic chi-square distribution under the null hypothesis when it is applied to residuals of an estimated GARCH model. Following Li and Mak [5], Li and Yu [6] derived a portmanteau test for the goodness-of-fit of the fitted ACD model when the errors follow the exponential distribution.

In this paper, we consider a portmanteau test for checking the adequacy of the fitted ACD model when the errors have a Weibull distribution. This paper has similarities to [6] since the two papers both follow the approach by Li and Mak [5] to construct the portmanteau test statistic. Besides the difference in the distribution of the error term, the functional form of the ACD model in the present paper is more general than that of [6], because the latter only discusses the ACD model with an ARCH-like form of the conditional mean duration.

The remainder of this paper is organized as follows. Section 2 presents the portmanteau test for the Weibull ACD model estimated by the maximum likelihood method. In Sect. 3, two Monte Carlo simulations are performed to study the finite-sample performance of the diagnostic tool and an illustrative example is reported to demonstrate its usefulness.

2 A Portmanteau Test

2.1 Basic Definitions and the ML Estimation

Consider the autoregressive conditional duration (ACD) model,

$$x_i = \psi_i \varepsilon_i, \qquad \psi_i = \omega + \sum_{j=1}^{p} \alpha_j x_{i-j} + \sum_{j=1}^{q} \beta_j \psi_{i-j}, \tag{1}$$

where $t_0 < t_1 < \cdots < t_n < \cdots$ are arrival times, $x_i = t_i - t_{i-1}$ is an interval, $\omega > 0$, $\alpha_j \geq 0, \beta_j \geq 0$, and the innovations $\{\varepsilon_t\}$ are identically and independently distributed $(i.i.d.)$ nonnegative random variables with mean one [3].

For ACD model at (1), we assume that the innovation ε_i has the density of a standardized Weibull distribution,

428552_1_En_4_Chapter ☑ TYPESET ☐ DISK ☐ LE ☑ CP Disp.:29/8/2016 Pages: 9 Layout: T1-Standard

$$f_\gamma(x) = \gamma c_\gamma x^{\gamma-1} \exp\{-c_\gamma x^\gamma\}, \qquad x \geq 0,$$

where $c_\gamma = [\Gamma(1+\gamma^{-1})]^\gamma$, $\Gamma(\cdot)$ is the Gamma function, and $E(\varepsilon_i) = 1$. The Weibull distribution has a decreasing (increasing) hazard function if $\gamma < 1$ ($\gamma > 1$) and reduces to the standard exponential distribution if $\gamma = 1$. We denote this model by WACD(p, q) in this paper.

Let $\alpha = (\alpha_1, \ldots, \alpha_p)'$, $\beta = (\beta_1, \ldots, \beta_q)'$ and $\theta = (\omega, \boldsymbol{\alpha}', \boldsymbol{\beta}')'$. Denote by $\boldsymbol{\lambda} = (\gamma, \boldsymbol{\theta}')'$ the parameter vector of the Weibull ACD model, and its true value $\boldsymbol{\lambda}_0 = (\gamma_0, \boldsymbol{\theta}_0')'$ is an interior point of a compact set $\Lambda \subset \mathbb{R}^{p+q+2}$. The following assumption gives some constraints on the parameter space Λ.

Assumption 1 $\omega > 0$, $\alpha_j > 0$ for $1 \leq j \leq p$, $\beta_j > 0$ for $1 \leq j \leq q$, $\sum_{j=1}^p \alpha_j + \sum_{j=1}^q \beta_j < 1$, and Polynomials $\sum_{j=1}^p \alpha_j x^j$ and $1 - \sum_{j=1}^q \beta_j x^j$ have no common root.

Given nonnegative observations $x_1, \ldots, x_n$, the log-likelihood function of the Weibull ACD model is

$$L_n(\boldsymbol{\lambda}) = \sum_{i=1}^n \left\{\log f_\gamma\left(\frac{x_i}{\psi_i(\boldsymbol{\theta})}\right) - \log \psi_i(\boldsymbol{\theta})\right\}$$
$$= \sum_{i=1}^n \left\{-\gamma \log[\psi_i(\boldsymbol{\theta})] - c_\gamma \left[\frac{x_i}{\psi_i(\boldsymbol{\theta})}\right]^\gamma\right\} + (\gamma - 1)\sum_{i=1}^n \log(x_i) + n \log(\gamma \cdot c_\gamma).$$

Note that the above functions all depend on unobservable values of x_i with $i \leq 0$, and some initial values are hence needed for $x_0, x_{-1}, \ldots, x_{1-p}$ and $\psi_0(\boldsymbol{\theta}), \psi_{-1}(\boldsymbol{\theta}), \ldots, \psi_{1-q}(\boldsymbol{\theta})$. We simply set them to be $\bar{x} = n^{-1}\sum_{i=1}^n x_i$, and denote the corresponding functions respectively by $\widetilde{\psi}_i(\boldsymbol{\theta})$ and $\widetilde{L}_n(\boldsymbol{\lambda})$. Thus, the MLE can be defined as

$$\widetilde{\boldsymbol{\lambda}}_n = (\widetilde{\gamma}_n, \widetilde{\boldsymbol{\theta}}_n')' = \underset{\boldsymbol{\lambda} \in \Lambda}{\operatorname{argmax}}\, \widetilde{L}_n(\boldsymbol{\lambda}).$$

Let

$$c_1(x, \gamma) = -\frac{\partial \log f_\gamma(x)}{\partial x} x - 1 = -\gamma(1 - c_\gamma x^\gamma)$$

and

$$c_2(x, \gamma) = \frac{\partial \log f_\gamma(x)}{\partial \gamma} = -c_\gamma x^\gamma \log(x) + \log(x) - c_\gamma' x^\gamma + \gamma^{-1} + c_\gamma'/c_\gamma,$$

where $c_\gamma' = \partial c_\gamma/\partial\gamma$. It can be verified that $E[c_1(\varepsilon_i, \gamma_0)] = 0$ and $E[c_2(\varepsilon_i, \gamma_0)] = 0$. Denote $\kappa_1 = \text{var}[c_1(\varepsilon_i, \gamma_0)]$, $\kappa_2 = \text{var}[c_2(\varepsilon_i, \gamma_0)]$, $\kappa_3 = \text{cov}[c_1(\varepsilon_i, \gamma_0), c_2(\varepsilon_i, \gamma_0)]$ and

$$\Sigma = \begin{pmatrix} \kappa_2 & \kappa_3 E[\psi_i^{-1}(\boldsymbol{\theta}_0)\partial\psi_i(\boldsymbol{\theta}_0)/\partial\boldsymbol{\theta}'] \\ \kappa_3 E[\psi_i^{-1}(\boldsymbol{\theta}_0)\partial\psi_i(\boldsymbol{\theta}_0)/\partial\boldsymbol{\theta}] & \kappa_1 E\{\psi_i^{-2}(\boldsymbol{\theta}_0)[\partial\psi_i(\boldsymbol{\theta}_0)/\partial\boldsymbol{\theta}][\partial\psi_i(\boldsymbol{\theta}_0)/\partial\boldsymbol{\theta}']\} \end{pmatrix}.$$

428552_1_En_4_Chapter ☑ TYPESET ☐ DISK ☐ LE ☑ CP Disp.:29/8/2016 Pages: 9 Layout: T1-Standard

If Assumption 1 holds, then $\widetilde{\boldsymbol{\lambda}}_n$ converges to $\boldsymbol{\lambda}_0$ in almost surely sense as $n \to \infty$, and $\sqrt{n}(\widetilde{\boldsymbol{\lambda}}_n - \boldsymbol{\lambda}_0) \to_d N(0, \Sigma^{-1})$ as $n \to \infty$; see Engle and Russell [3] and Francq and Zakoian [4].

Denote by $\{\widetilde{\varepsilon}_i\}$ the residual sequence from the fitted Weibull ACD model, where $\widetilde{\varepsilon}_i = x_i/\widetilde{\psi}_i(\widetilde{\boldsymbol{\theta}}_n)$. For the quantities in the information matrix Σ, κ_1, κ_2, κ_3, $E[\psi_i^{-1}(\boldsymbol{\theta}_0)\partial\psi_i(\boldsymbol{\theta}_0)/\partial\boldsymbol{\theta}]$, and $E[\psi_i^{-2}(\boldsymbol{\theta}_0)(\partial\psi_i(\boldsymbol{\theta}_0)/\partial\boldsymbol{\theta})(\partial\psi_i(\boldsymbol{\theta}_0)/\partial\boldsymbol{\theta}')]$, we can estimate them respectively by

$$\widetilde{\kappa}_1 = \frac{1}{n}\sum_{i=1}^n [c_1(\widetilde{\varepsilon}_i, \widetilde{\gamma}_n)]^2, \quad \widetilde{\kappa}_2 = \frac{1}{n}\sum_{i=1}^n [c_2(\widetilde{\varepsilon}_i, \widetilde{\gamma}_n)]^2, \quad \widetilde{\kappa}_3 = \frac{1}{n}\sum_{i=1}^n c_1(\widetilde{\varepsilon}_i, \widetilde{\gamma}_n)c_2(\widetilde{\varepsilon}_i, \widetilde{\gamma}_n),$$

$$\frac{1}{n}\sum_{i=1}^n \frac{1}{\widetilde{\psi}_i(\widetilde{\boldsymbol{\theta}}_n)}\frac{\partial\widetilde{\psi}_i(\widetilde{\boldsymbol{\theta}}_n)}{\partial\boldsymbol{\theta}} \quad \text{and} \quad \frac{1}{n}\sum_{i=1}^n \frac{1}{\widetilde{\psi}_i^2(\widetilde{\boldsymbol{\theta}}_n)}\frac{\partial\widetilde{\psi}_i(\widetilde{\boldsymbol{\theta}}_n)}{\partial\boldsymbol{\theta}}\frac{\partial\widetilde{\psi}_i(\widetilde{\boldsymbol{\theta}}_n)}{\partial\boldsymbol{\theta}'}.$$

The above estimators are all consistent, and hence a consistent estimator of the information matrix Σ. Moreover,

$$\sqrt{n}(\widetilde{\boldsymbol{\theta}}_n - \boldsymbol{\theta}_0) \to_d N(0, \Sigma_1^{-1}) \quad \text{as } n \to \infty, \tag{2}$$

where

$$\Sigma_1 = \kappa_1 \cdot E\left[\frac{1}{\psi_i^2(\boldsymbol{\theta}_0)}\frac{\partial\psi_i(\boldsymbol{\theta}_0)}{\partial\boldsymbol{\theta}}\frac{\partial\psi_i(\boldsymbol{\theta}_0)}{\partial\boldsymbol{\theta}'}\right] - \frac{\kappa_3^2}{\kappa_2}\cdot E\left[\frac{1}{\psi_i(\boldsymbol{\theta}_0)}\frac{\partial\psi_i(\boldsymbol{\theta}_0)}{\partial\boldsymbol{\theta}}\right]E\left[\frac{1}{\psi_i(\boldsymbol{\theta}_0)}\frac{\partial\psi_i(\boldsymbol{\theta}_0)}{\partial\boldsymbol{\theta}'}\right].$$

2.2 The Main Result

This subsection derives asymptotic distributions of the residual autocorrelations from the estimated Weibull ACD model, and hence a portmanteau test for checking the adequacy of this model. Note that the residuals are nonnegative, and the residual autocorrelations here are also the absolute residual autocorrelations.

Without confusion, we denote $\widetilde{\psi}_i(\widetilde{\boldsymbol{\theta}}_n)$ and $\psi_i(\boldsymbol{\theta}_0)$ respectively by $\widetilde{\psi}_i$ and ψ_i for simplicity. Consider the residual sequence $\{\widetilde{\varepsilon}_i\}$ with $\widetilde{\varepsilon}_i = x_i/\widetilde{\psi}_i$. Note that $n^{-1}\sum_{i=1}^n \widetilde{\varepsilon}_i = 1 + o_p(1)$ and then, for a positive integer k, the lag-k residual autocorrelation can be defined as

$$\widetilde{r}_k = \frac{\sum_{i=k+1}^n (\widetilde{\varepsilon}_i - 1)(\widetilde{\varepsilon}_{i-k} - 1)}{\sum_{i=1}^n (\widetilde{\varepsilon}_i - 1)^2}.$$

We next consider the asymptotic distributions of the first K residual autocorrelations, $\widetilde{R} = (\widetilde{r}_1, \ldots, \widetilde{r}_K)'$, where K is a predetermined positive integer.

Denote $\widetilde{\psi}_i(\widetilde{\boldsymbol{\theta}}_n)$ and $\psi_i(\boldsymbol{\theta}_0)$ respectively by $\widetilde{\psi}_i$ and ψ_i, and let $\widetilde{\varepsilon}_i = x_i/\widetilde{\psi}_i$. Let $\widetilde{C} = (\widetilde{C}_1, \ldots, \widetilde{C}_K)'$ and $C = (C_1, \ldots, C_K)'$, where

$$\widetilde{C}_k = \frac{1}{n}\sum_{i=k+1}^{n}(\widetilde{\varepsilon}_i - 1)(\widetilde{\varepsilon}_{i-k} - 1) \quad \text{and} \quad C_k = \frac{1}{n}\sum_{i=k+1}^{n}(\varepsilon_i - 1)(\varepsilon_{i-k} - 1).$$

By the $\sqrt{n}$-consistency of $\widetilde{\boldsymbol{\theta}}_n$ at (2) and the ergodic theorem, it follows that $n^{-1}\sum_{i=1}^{n}(\widetilde{\varepsilon}_i - 1)^2 = \sigma_{\gamma_0}^2 + o_p(1)$, where $\sigma_{\gamma_0}^2 = \text{var}(\varepsilon_i)$, and thus it suffices to derive the asymptotic distribution of $\widetilde{C}$.

By the Taylor expansion, it holds that

$$\widetilde{C} = C + H'(\widetilde{\boldsymbol{\theta}}_n - \boldsymbol{\theta}_0) + o_p(n^{-1/2}), \tag{3}$$

where $H = (H_1, \ldots, H_K)$ with $H_k = -E[\psi_i^{-1}(\varepsilon_{i-k} - 1)\partial\psi_i/\partial\boldsymbol{\theta}]$. Moreover,

$$\sqrt{n}(\widetilde{\boldsymbol{\theta}}_n - \boldsymbol{\theta}_0) = A\Sigma^{-1} \cdot \frac{1}{\sqrt{n}}\sum_{i=1}^{n}\left[c_2(\varepsilon_i, \gamma_0), \frac{c_1(\varepsilon_i, \gamma_0)}{\psi_i}\frac{\partial\psi_i}{\partial\boldsymbol{\theta}'}\right]' + o_p(1), \tag{4}$$

where the $c_j(\varepsilon_i, \gamma_0)$ is as defined in Sect. 2.1, and the matrix $A = (0, \mathbf{I})$ with $\mathbf{I}$ being the $(p + q + 1)$-dimensional identity matrix. Note that $E[\varepsilon_i c_2(\varepsilon_i, \gamma_0)] = 0$ and $E[\varepsilon_i c_1(\varepsilon_i, \gamma_0)] = 1$. By (3), (4), the central limit theorem and the Cramér-Wold device, it follows that

$$\sqrt{n}\widetilde{R} \to_d N(0, \Omega) \quad \text{as } n \to \infty,$$

where $\Omega = \mathbf{I} - \sigma_{\gamma_0}^{-4} H'\Sigma_1^{-1}H$, $\sigma_{\gamma_0}^2 = \text{var}(\varepsilon_i)$, $H = (H_1, \ldots, H_K)$ with $H_k = -E[\psi_i^{-1}(\varepsilon_{i-k} - 1)\partial\psi_i/\partial\boldsymbol{\theta}]$, and Σ_1 is as defined in Sect. 2.1.

Let $\widetilde{\sigma}_{\gamma_0}^2 = n^{-1}\sum_{i=1}^{n}(\widetilde{\varepsilon}_i - 1)^2$, $\widetilde{H}_k = -n^{-1}\sum_{i=1}^{n}\widetilde{\psi}_i^{-1}(\widetilde{\varepsilon}_{i-k} - 1)\partial\widetilde{\psi}_i/\partial\boldsymbol{\theta}$ and $\widetilde{H} = (\widetilde{H}_1, \ldots, \widetilde{H}_K)$. Then we have $\widetilde{H} = H + o_p(1)$ and hence a consistent estimator of Ω can be constructed, denoted by $\widetilde{\Omega}$. Let $\widetilde{\Omega}_{kk}$ be the diagonal elements of $\widetilde{\Omega}$, for $1 \le k \le K$. We therefore can check the significance of $\widetilde{r}_k$ by comparing its absolute value with $1.96\sqrt{\widetilde{\Omega}_{kk}/n}$, where the significance level is 5 %.

To check the significance of $\widetilde{R} = (\widetilde{r}_1, \ldots, \widetilde{r}_K)'$ jointly, we can construct a portmanteau test statistic,

$$Q(K) = n\widetilde{R}'\widetilde{\Omega}^{-1}\widetilde{R},$$

and it will be asymptotically distributed as χ_K^2, the chi-square distribution with K degrees of freedom.

3 Numerical Studies

3.1 Simulation Experiments

This subsection conducts two Monte Carlo simulation experiments to check the finite-sample performance of the proposed portmanteau test in the previous section.

428552_1_En_4_Chapter ☑ TYPESET ☐ DISK ☐ LE ☑ CP Disp.:29/8/2016 Pages: 9 Layout: T1-Standard

The first experiment evaluates the sample approximation for the asymptotic variance of residual autocorrelations Ω, and the data generating process is

$$x_i = \psi_i \varepsilon_i, \qquad \psi_i = 0.1 + \alpha x_{i-1} + \beta \psi_{i-1},$$

where ε_i follows the standardized Weibull distribution with the parameter of γ. We consider $\gamma = 0.8$ and 1.2, corresponding to a heavy-tailed distribution and a light-tailed one, and $(\alpha, \beta)' = (0.2, 0.6)'$ and $(0.4, 0.5)'$. The sample size is set to $n = 200$, 500 or 1000, and there are 1000 replications for each sample size. As shown in Table 1, the asymptotic standard deviations (ASDs) of the residual autocorrelations at lags 2, 4 and 6 are close to their corresponding empirical standard deviations (ESDs) when the sample size is as small as $n = 500$.

In the second experiment, we check the size and power of the proposed portmanteau test $Q(K)$ using the data generating process,

$$x_i = \psi_i \varepsilon_i, \qquad \psi_i = 0.1 + 0.3x_{i-1} + \alpha_2 x_{i-2} + 0.3\psi_{i-1},$$

where $\alpha_2 = 0$, 0.15 or 0.3, and ε_i follows the standardized Weibull distribution with $\gamma = 0.8$ or 1.2. All the other settings are preserved from the previous experiment. We fit the model of orders (1, 1) to the generated data; hence, the case with $\alpha_2 = 0$ corresponds to the size and those with $\alpha_2 > 0$ to the power. The rejection rates of test statistic $Q(K)$ with $K = 6$ are given in Table 2. For comparison, the corresponding rejection rates of the Ljung–Box statistics for the residual series and its squared process are also reported, denoted by $Q_1^*(K)$ and $Q_2^*(K)$. The critical value is the upper 5th percentile of the χ_6^2 distribution for all these tests. As shown in the table,

Table 1 Empirical standard deviations (ESD) and asymptotic standard deviations (ASD) of residual autocorrelations at lags 2, 4 and 6

	n		$\boldsymbol{\theta} = (0.1, 0.2, 0.6)'$			$\boldsymbol{\theta} = (0.1, 0.4, 0.5)'$		
			2	4	6	2	4	6
$\gamma = 0.8$	200	ESD	0.1025	0.1061	0.1065	0.0610	0.0660	0.0635
		ASD	0.0605	0.0655	0.0673	0.0625	0.0658	0.0675
	500	ESD	0.0402	0.0415	0.0431	0.0389	0.0419	0.0416
		ASD	0.0387	0.0411	0.0424	0.0402	0.0418	0.0427
	1000	ESD	0.0284	0.0289	0.0301	0.0280	0.0297	0.0305
		ASD	0.0277	0.0291	0.0298	0.0285	0.0297	0.0301
$\gamma = 1.2$	200	ESD	0.0847	0.0862	0.0889	0.0632	0.0656	0.0658
		ASD	0.0604	0.0652	0.0673	0.0629	0.0659	0.0674
	500	ESD	0.0386	0.0414	0.0421	0.0395	0.0433	0.0410
		ASD	0.0387	0.0409	0.0422	0.0401	0.0418	0.0426
	1000	ESD	0.0277	0.0290	0.0296	0.0276	0.0301	0.0292
		ASD	0.0276	0.0289	0.0297	0.0284	0.0296	0.0301

Table 2 Rejection rates of the test statistics $Q(K)$, $Q_1^*(K)$ and $Q_2^*(K)$ with $K = 6$ and $\gamma = 0.8$ or 1.2

	n	$\alpha_2 = 0$		$\alpha_2 = 0.15$		$\alpha_2 = 0.3$	
		0.8	1.2	0.8	1.2	0.8	1.2
$Q(K)$	200	0.101	0.107	0.110	0.131	0.196	0.305
	500	0.085	0.089	0.147	0.172	0.414	0.633
	1000	0.080	0.092	0.205	0.314	0.709	0.934
$Q_1^*(K)$	200	0.021	0.022	0.041	0.052	0.133	0.207
	500	0.013	0.018	0.076	0.082	0.329	0.558
	1000	0.016	0.008	0.115	0.203	0.639	0.899
$Q_2^*(K)$	200	0.046	0.022	0.059	0.048	0.084	0.139
	500	0.051	0.024	0.080	0.072	0.149	0.314
	1000	0.052	0.022	0.088	0.135	0.209	0.617

the test $Q(K)$ is oversized when $n = 1000$, while the other two tests are largely undersized for some γ. Furthermore, we found that increasing the sample size to 9000 could result in $Q(K)$ having sizes of 0.058 and 0.053 for $\gamma = 0.8$ and 1.2, while the sizes of the other two tests do not become closer to the nominal value even for very large n. For the power simulations, it can be seen clearly that $Q(K)$ is the most powerful test among the three and $Q_2^*(K)$ is the least powerful one. Moreover, the powers are interestingly observed to have smaller values when the generated data are heavy-tailed ($\gamma = 0.8$).

3.2 An Empirical Example

As an illustrative example, this subsection considers the trade durations of the US IBM stock on fifteen consecutive trading days starting from November 1, 1990. The data are truncated from a larger data set which consists of the diurnally adjusted IBM trade durations data from November 1, 1990, to January 31, 1991, adjusted

Table 3 Model diagnostic checking results for the adjusted durations for IBM stock traded in first fifteen trading days of November 1990: p values for $Q(K)$, $Q_1^*(K)$ and $Q_2^*(K)$ with $K = 6$, 12 and 18, at the 5 % significance level

K	$q = 1$			$q = 2$			$q = 3$		
	$Q(K)$	$Q_1^*(K)$	$Q_2^*(K)$	$Q(K)$	$Q_1^*(K)$	$Q_2^*(K)$	$Q(K)$	$Q_1^*(K)$	$Q_2^*(K)$
6	0.0081	0.0123	0.4827	0.0560	0.0938	0.3778	0.3915	0.5010	0.5172
12	0.0225	0.0233	0.4313	0.1157	0.1372	0.3890	0.4933	0.5427	0.5315
18	0.0012	0.0022	0.0723	0.0116	0.0190	0.0727	0.0815	0.1200	0.1211

and analyzed by Tsay [9, Chap. 5]. Focusing on positive durations, we have 12,532 diurnally adjusted observations.

We consider the WACD(p, q) models with $p = 1$ and $q = 1, 2$ or 3. The major interest is on whether the models fit the data adequately. To this end, the p values for $Q(K)$, $Q_1^*(K)$ and $Q_2^*(K)$ with $K = 6$, 12 and 18 at the 5 % significance level are reported in Table 3. It can be seen that the WACD(1, 3) model fits the data adequately according to all the test statistics. The fitted WACD(1, 1) model is clearly rejected by both $Q(K)$ and $Q_1^*(K)$ with $K = 6, 12$ and 18. For the fitted WACD(1, 2) model, both $Q(K)$ and $Q_1^*(K)$ suggest an adequate fit of the data with $K = 6$ or 12, but not with $K = 18$. While for the data, $Q(K)$ and $Q_1^*(K)$ always lead to the same conclusions, the fact that the p value for $Q(K)$ is always smaller than that for $Q_1^*(K)$ confirms that $Q(K)$ is more powerful than $Q_1^*(K)$. In contrast, $Q_2^*(K)$ fails to detect any inadequacy of the fitted WACD models.

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References

1. Bollerslev, T. (1986). Generalized autoregressive conditional heteroskedasticity. *Journal of Econometrics*, *31*, 307–327.
2. Engle, R. F. (1982). Autoregression conditional heteroscedasticity with estimates of the variance of U.K. inflation. *Econometrica*, *50*, 987–1008.
3. Engle, R. F., & Russell, J. R. (1998). Autoregressive conditional duration: A new model for irregularly spaced transaction data. *Econometrica*, *66*, 1127–1162.
4. Francq, C., & Zakoian, J. M. (2004). Maximum likelihood estimation of pure GARCH and ARMA-GARCH processes. *Bernoulli*, *10*, 605–637.
5. Li, W. K., & Mak, T. K. (1994). On the squared residual autocorrelations in non-linear time series with conditoinal heteroskedasticity. *Journal of Time Series Analysis*, *15*, 627–636.
6. Li, W. K., & Yu, P. L. H. (2003). On the residual autocorrelation of the autoregressive conditional duration model. *Economic Letters*, *79*, 169–175.
7. Ljung, G. M., & Box, G. E. P. (1978). On a measure of lack of fit in time series models. *Biometrika*, *65*, 297–303.
8. McLeod, A. I., & Li, W. K. (1983). Diagnostic checking ARMA time series models using squared residual autocorrelations. *Journal of Time Series Analysis*, *4*, 269–273.
9. Tsay, R. S. (2010). *Analysis of financial time series* (3rd ed.). New York: Wiley.